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DETERMINING THE STRESS STATE OF A STRETCHED ROD FROM ITS MEASURED MAGNETIC CHARACTERISTICS

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Stress–strain diagrams and coercive force–strain diagrams for carbon steel samples under uniaxial tension were obtained experimentally during loading and after unloading. These diagrams were used to develop a procedure for determining the stress acting in structural members from the value of the coercive force in the loaded state. A method is proposed to estimate the previous maximum stress from the values of the coercive force measured in the unloaded state.

Key words: *witness sample, coercive force, stress state.*

Introduction. The relationship between the physicomechanical and magnetic properties of ferromagnets has been established experimentally. Many magnetic methods of structural observation and nondestructive control are based on the use of the coercive force H_C, which is the main characteristic of the magnetic hysteresis loop and, by the definition, does not depend on the geometrical dimensions of the sample. The coercive force is equal in value to the demagnetizing field that needs to be applied to a magnetized ferromagnet to reduce its magnetization to zero. The standard attachable coercive force meters are intended to measure this characteristic for various articles and structural members, including under field conditions.

As is known, the coercive force reacts to a change in the structural state of material, its chemical composition, and internal and external stresses. For example, the dependence of the coercive H_C on the grain size $d_{\rm y}$ is generally written as [1]

$$H_C \simeq A/d_{\rm y} + B,$$

where A and B are some numerical coefficients. This formula is similar to the well-known Hall–Petch relation between the yield point and grain size [2]

$$\sigma_{\rm y} = \sigma_0 + K_y d_{\rm y}^{-1/2},$$

where σ_0 is a parameter that characterizes the resistance of the crystal lattice to dislocation motion and K_y is a coefficient that describes the contribution of the grain boundaries to the hardening.

In practice, wide use is made of the dependence of the coercive forces on the volume fraction V of dispersed particles in the structure of material [3]:

$$H_C \simeq KV^n/M_s.$$

Here K is a numerical coefficient, M_s is the saturation magnetization of the material, and the exponent n can vary from 1/3 to 1, depending on the particle size. This formula is similar to the well-known dependence of the yield point on the volume fraction of dispersed particles

$$\sigma_{\rm y} \simeq G|\boldsymbol{b}|V^{1/3}/d,$$

where G is the shear modulus, $\boldsymbol{b}$ is the Burgers vector, and d is the particle size.

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The effect of the dislocation density N on the coercive force is described by the relation [4]

$$H_C \simeq \lambda_s G|\boldsymbol{b}|\sqrt{N}\,/M_s,$$

where λ_s is the saturation magnetic striction. There is a similar dependence of the yield point of material on dislocation density:

$$\sigma_{\rm y} \simeq a|\boldsymbol{b}|G\sqrt{N}$$

(a is a numerical coefficient).

The above dependences and data of various experimental studies suggest that the effects of structural parameters on the magnetic and mechanical characteristics have many features in common and that there are stable correlations between the stress–strain state of an article and changes in its magnetic characteristics [5–7]. Similar conclusions are contained in a theoretical study [8] performed at the interface between continuum mechanics and solid-state physics, where the interaction between the lattice continuum (deformation carrier) and the magnetization field is described.

In the opinion of most researchers, in the elastic region of loading, changes in the coercive forces are due primarily to the formation of the magnetic texture of stresses and have a reversible nature. In the plastic region of loading, where the density of dislocations and other structural defects increase considerably, the coercive force, as a rule, increases irreversibly. It should be noted that mathematical apparatus for estimating the stress state of a solid from the results of magnetic measurements has not been developed. Nevertheless, in practice there is a need to determine the stress–strain state of structural members from their magnetic characteristics and to develop corresponding methods.

In the present work, a comparison was made of stress–strain diagrams for steel with a change in its coercive force measured directly during the action of load P and after unloading. Uniaxial tension tests were performed with samples (working area of $2 \times 20 \times 90$ mm) cut out along the rolling direction from a hot-rolled sheet of commercial St. 45 steel melt. The samples were magnetized along the tension axis. The experimental technique and the parameters of the setup for measuring the magnetic characteristics of steel under loading conditions are described in [7].

Formulation of the Problem and Basic Equations. To take into account the real properties of materials in calculating stress states of structural members, it is necessary to find stress–strain relations in such a form that the elasticity equations are satisfied and the calculation is as simple as possible. For the case of loading of material under conditions of isothermal plastic deformation, its state can be defined by a set of parameters σ_{ij}, ε_{ij}, q_i, and k_i (σ_{ij} and ε_{ij} are the stress and strain tensor components, and q_i are hardening parameters; and k_i are material constants) [9].

Plastic theory [10] assumes that, in simple loading, where all loading forces P increase in proportion to the same parameter (in this case, time), there exists the invariant relationship

$$\sigma_i = f(\varepsilon_i)\varepsilon_i \tag{1}$$

(σ_i and ε_i are the stress and strain rates, respectively).

For elastic deformation, relation (1) becomes

$$\sigma_i = E\varepsilon_i,$$

where E is the elastic modulus.

Relation (1) is also valid for uniaxial loading, which was used in the tests described here. In engineering applications, the dependence $f(\varepsilon)$ can be approximated by the continuous bilinear straight line [9–12]:

$$f(\varepsilon) = \begin{cases} E, & \varepsilon_i < \varepsilon_{\rm y}, \\ E_c, & \varepsilon_i > \varepsilon_{\rm y} \end{cases}$$

(E_c is the elastic modulus in the hardening region). There are various analytical approximations of tension curves $f(\varepsilon)$ that take into account phase transitions during plastic flow, the evolution of the dislocation structure, etc. [9]. However, there are no universal analytical dependences that accurately describe the yield "tooth," the yield plateau, and the region of developed plastic deformation.

To determine the stress state of a structural member under axial tension from measurements of the coercive forces, it is necessary to know the stress–strain relations $\sigma(\varepsilon)$ and the coercive force–strain $H_C(\varepsilon)$ relations for a witness sample. It should be noted that the geometrical dimensions of the witness sample are of no significance because the coercive force generally does not depend on the region and cross-sectional shape of the ferromagnet.

For a sample in uniaxial tension under simple loading, we experimentally construct the dependence $\sigma(\varepsilon)$ for n deformation steps. Thus, we have

$$\sigma = f_1(\varepsilon), \qquad d\sigma > 0. \tag{2}$$

Determining the coercive forces in each loading step, we find the dependence

$$H_C = f_2(\varepsilon), \qquad P > 0. \tag{3}$$

Unloading the sample after each loading step and determining the residual strain and the coercive forces H_C^* in the unloaded state, we obtain the relation

$$H_C^* = f_3(\varepsilon_{\text{resid}}), \qquad P = 0. \tag{4}$$

Approximating the functions f_1, f_2, and f_3 in the form of n piecewise-linear dependences with arbitrary steps in strain ε_k $(k = 0, 1, 2, \ldots, n)$, we determine the values of the stress and coercive force in the loaded and unloaded states in the first region $(\varepsilon_0 \leqslant \varepsilon \leqslant \varepsilon_1)$, i.e., for $k = 1$:

$$\sigma = \frac{\sigma_k - \sigma_{k-1}}{\varepsilon_k - \varepsilon_{k-1}}\,(\varepsilon - \varepsilon_{k-1}); \tag{5}$$

$$H_C = H_{C0} + \frac{H_{Ck} - H_{Ck-1}}{\varepsilon_k - \varepsilon_{k-1}}\,(\varepsilon - \varepsilon_{k-1}); \tag{6}$$

$$H_C^* = H_{C0}^* + \frac{H_{Ck}^* - H_{Ck-1}^*}{\varepsilon_k - \varepsilon_{k-1}}\,(\varepsilon - \varepsilon_{\text{elastic}} - \varepsilon_{k-1}). \tag{7}$$

Here σ_k and H_{Ck} are the stress and coercive force under deformation ε_k, $H_{C0} = H_{C0}^*$ is the coercive forces before the test of the sample, which characterizes the material in the initial state, and $\varepsilon_{\text{elastic}}$ is the elastic strain.

In the given region, the slopes of the functions f_1, f_2, and f_3 are constant:

$$\frac{\sigma_1 - \sigma_0}{\varepsilon_1 - \varepsilon_0} = \frac{\Delta\sigma_1}{\Delta\varepsilon_1} = \tan\,\alpha_1 = \text{const},$$

$$\frac{H_{C1} - H_{C0}}{\varepsilon_1 - \varepsilon_0} = \frac{\Delta H_{C1}}{\Delta\varepsilon_1} = \tan\,\beta_1 = \text{const}, \qquad \frac{H_{C1}^* - H_{C0}^*}{\varepsilon_1 - \varepsilon_0} = \frac{\Delta H_{C1}^*}{\Delta\varepsilon_1} = \tan\,\beta_1^* = \text{const}.$$

Therefore, for this region, it is possible to introduce the weight coefficients

$$C_1 = \frac{\tan\,\alpha_1}{\tan\,\beta_1} = \frac{\Delta\sigma_1}{\Delta H_{C1}}; \tag{8}$$

$$C_1^* = \frac{\tan\,\alpha_1}{\tan\,\beta_1^*} = \frac{\Delta\sigma_1}{\Delta H_{C1}^*}. \tag{9}$$

Thus, in view of relations (5), (6), and (8), the stresses arising in the first loading step can be determined from the value of the coercive force:

$$\sigma = C_1(H_C - H_{C0}). \tag{10}$$

Here $H_{C0} \leqslant H_C \leqslant H_{C1}$ for $dH_C > 0$ and $H_{C0} \geqslant H_C \geqslant H_{C1}$ for $dH_C < 0$.

From relations (5), (7), and (9), ignoring the quantity $\varepsilon_{\text{elastic}}$ because of its smallness, we determine the stress in the sample in the first loading step from the value of the coercive force measured in the sample after unloading:

$$\sigma = C_1^*(H_C^* - H_{C0}^*). \tag{11}$$

Since relations (10) and (11) are similar in form, the relationship between the stress and the coercive force will be further determined for the loaded state, and the obtained dependences will be extrapolated to the relations with the coercive force measured in the unloaded sample.

The stress and coercive forces in the loaded state in the second region $(\varepsilon_1 < \varepsilon \leqslant \varepsilon_2)$, i.e., for $k = 2$, are determined similarly:

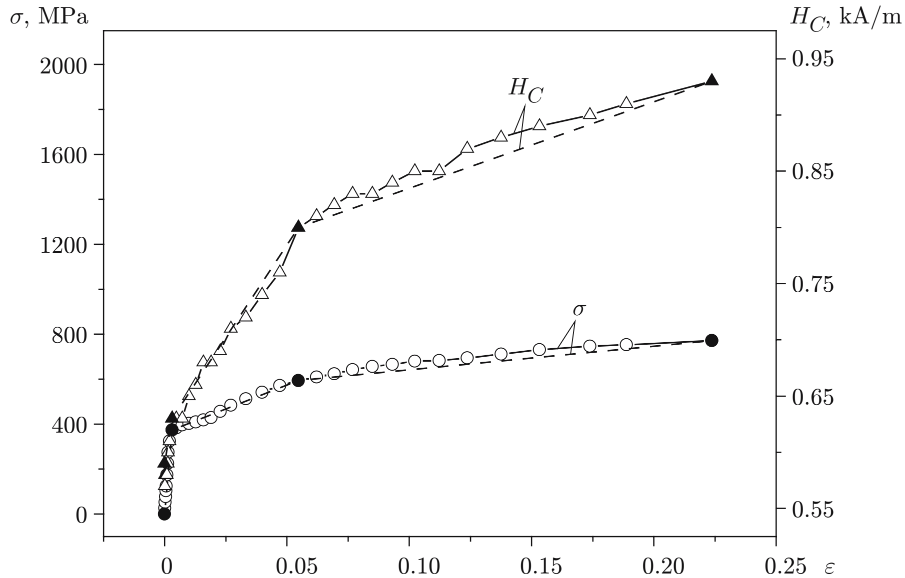

Fig. 1. Stress–strain $\sigma(\varepsilon)$ diagram and a curve of coercive forces versus strain $H_C(\varepsilon)$ in the unloading state for hot-rolled St. 45 steel: solid curves correspond to experimental data, and dashed curves to a piecewise linear approximation ($n = 3$).

$$\sigma = \Delta\sigma_{k-1} + \frac{\sigma_k - \sigma_{k-1}}{\varepsilon_k - \varepsilon_{k-1}}\,(\varepsilon - \varepsilon_{k-1}); \tag{12}$$

$$H_C = H_{C0} + \Delta H_{Ck-1} + \frac{H_{Ck} - H_{Ck-1}}{\varepsilon_k - \varepsilon_{k-1}}\,(\varepsilon - \varepsilon_{k-1}). \tag{13}$$

For the second region, introducing the coefficients

$$C_k = \frac{\sigma_k - \sigma_{k-1}}{H_{Ck} - H_{Ck-1}} = \frac{\Delta\sigma_k}{\Delta H_{Ck}}, \tag{14}$$

and using Eqs. (12) and (13), we determine the stresses from the value of the coercive force:

$$\sigma = C_2(H_C - H_{C0}) + (C_1 - C_2)\Delta H_{C1}.$$

Here $H_{C1} < H_C \leqslant H_{C2}$ for $dH_C > 0$ and $H_{C1} > H_C \geqslant H_{C2}$ for $dH_C < 0$.

Thus, we can determine the stress arising in any kth loading region:

$$\sigma = C_k(H_C - H_{C0}) + \sum_{i=1}^{k-1} (C_i - C_k)\Delta H_{Ci}. \tag{15}$$

Here $H_{C(k-1)} < H_C \leqslant H_{C(k)}$ for $dH_C > 0$ and $H_{C(k-1)} > H_C \geqslant H_{C(k)}$ for $dH_C < 0$.

Representing Eq. (15) in the form

$$\sigma = C_k H_C + D_k, \tag{16}$$

where

$$D_k = \sum_{i=1}^{k-1} (C_i - C_k)\Delta H_{Ci} - C_k H_{C0}, \tag{17}$$

it is easy to see that, in any kth region, the stress is defined by a linear relation which include the current value of the coercive forces and two constants C_k and D_k dependent on the physicomechanical properties of the particular ferromagnet.

Results of Approximation of Experimental Curves. Figure 1 shows experimental curves (2) and (3) for $n = 34$ loading step. From Fig. 1 it follows that, in estimating tensile stresses of structural members from the material studied (St. 45 steel), one can use a piecewise-linear ($n = 3$) approximation of the dependences $\sigma(\varepsilon)$ and

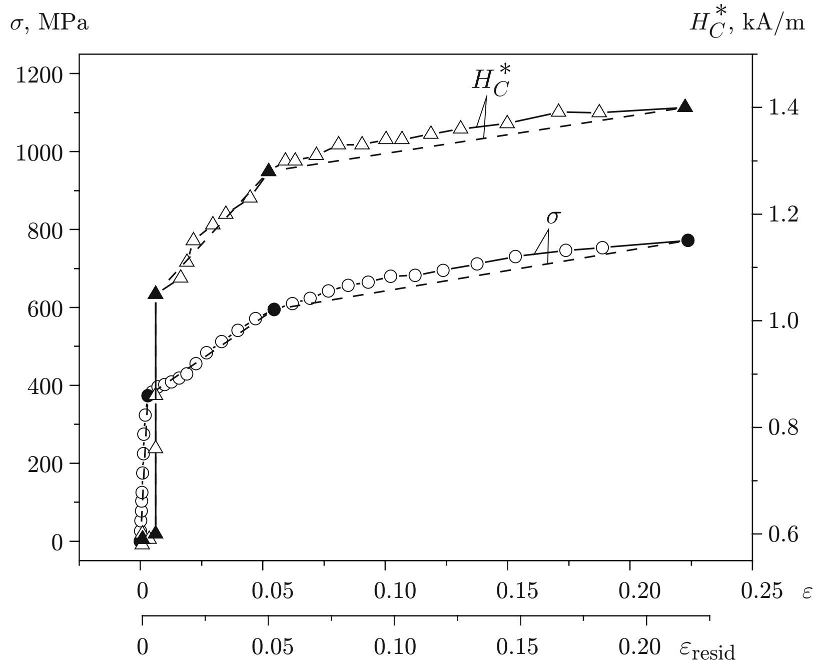

Fig. 2. Stress–strain $\sigma(\varepsilon)$ diagram and a curve of the coercive forces versus residual strain $H_C^*(\varepsilon_{\text{resid}})$ in the unloaded state for hot-rolled St. 45 steel: solid curves refer to the experimental data, and dashed curves to a piecewise-linear approximation ($n = 4$).

$H_C(\varepsilon)$. In the curve of $\sigma(\varepsilon)$, the first region corresponds to the elastic stage of deformation, and the second and third regions correspond to the stages of plastic deformation with various degrees of deformation hardening; the dependence $H_C(\varepsilon)$ is similar to the dependence $\sigma(\varepsilon)$.

Choosing four characteristic experimental points for approximating the functions f_1 and f_2: 1) $\varepsilon_0 = 0$, $\sigma = 0$, and $H_{C0} = 0.59$ kA/m; 2) $\varepsilon_1 = 0.003$, $\sigma_1 = 374$ MPa, and $H_{C1} = 0.63$ kA/m; 3) $\varepsilon_2 = 0.05$, $\sigma_2 = 594$ MPa, and $H_{C2} = 0.80$ kA/m; 4) $\varepsilon_3 = 0.22$, $\sigma_3 = 772$ MPa, and $H_{C3} = 0.93$ kA/m and calculating the coefficients by formulas (14) and (17), we find three linear dependences of the form of (16) for determining stresses from values of the coercive force H_C:

$$\sigma = \begin{cases} 9343H_C - 5512, & 0.59 \leqslant H_C \leqslant 0.63, \\ 1298H_C - 444, & 0.63 < H_C \leqslant 0.80, \\ 1367H_C - 499, & 0.80 < H_C \leqslant 0.93. \end{cases} \tag{18}$$

Thus, the obtained linear dependences (18) for the witness sample of St. 45 steel (the calculation error is less than 3%) can be used to determine the stress level in structural members of St. 45 steel in axial tension from values of the coercive forces H_C measured in the loading state.

As noted above, because relations (10) and (11) formally coincide, all calculations performed in the derivation of the dependence $\sigma = f(H_C)$ are valid for the determination of the coercive forces in the unloaded state H_C^*. Hence, the relationship between the stress arising in the sample and the value of the coercive forces H_C^* measured in the unloaded sample is similar to relation (16):

$$\sigma = C_k^* H_C^* + D_k^*.$$

Here

$$D_k^* = \sum_{i=1}^{k-1} (C_i^* - C_k^*)\,\Delta H_{Ci}^* - C_k^* H_{C0}^*. \tag{19}$$

Figure 2 shows experimental curves of (2) and (4) for $n = 34$ loading steps. From Fig. 2, it follows that, in estimating tensile stresses of structural members from the material studied (St. 45 steel), one can use a piecewise-linear ($n = 4$) approximation of the dependences $\sigma(\varepsilon)$ and $H_C^*(\varepsilon)$. In the dependence $H_C^*(\varepsilon)$, the first region corresponds to the elastic stage of deformation, the second to a sharp increase in the coercive forces at the yield point, and the third and fourth regions correspond to deformation with various degrees of hardening.

Choosing five characteristic experimental points for approximating the functions f_1 and f_3: 1) $\varepsilon_0 = 0$, $\sigma_0 = 0$, and $H^*_{C0} = 0.59$ kA /m; 2) $\varepsilon_1 = 0.003$, $\sigma_1 = 374$ MPa, and $H^*_{C1} = 0.6$ kA/m; 3) $\varepsilon_2 = 0.003$, $\sigma_2 = 374$ MPa, $H^*_{C2} = 1.05$ kA/m; 4) $\varepsilon_3 = 0.05$, $\sigma_3 = 594$ MPa, and $H^*_{C3} = 1.31$ kA/m; 5) $\varepsilon_4 = 0.22$, $\sigma_4 = 772$ MPa, and $H^*_{C4} = 1.4$ kA/m and calculating the coefficients by formulas (19), we obtain four linear dependences to determine stresses from the values of the coercive force H^*_C:

$$\sigma = \begin{cases} 37\,400 H^*_C - 22{,}066, & 0.59 \leqslant H^*_C \leqslant 0.60, \\ 374, & 0.60 < H^*_C \leqslant 1.05, \\ 846 H^*_C - 514, & 1.05 < H^*_C \leqslant 1.31, \\ 1978 H^*_C - 1997, & 1.31 < H^*_C \leqslant 1.40. \end{cases} \tag{20}$$

Thus, the linear dependences (20) obtained for the St. 45 steel witness sample (the calculation error is less than 3%) can be used to determine the maximum tensile stress in structural members from St. 45 steel under simple loading from results of measurements of their magnetic characteristic — the coercive forces H^*_C in the unloaded state.

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